

Functional inequality between Hessians

Michał Gaczkowski*

* Warsaw University of Technology, Poland
E-mail: Michal.Gaczkowski@pw.edu.pl

Tomasz Cieřlak, Mario Fuest, Karol Hajduk, and Mikořaj Sieręga recently showed in their paper "On the existence of global solutions for the 3D chemorepulsion system", that for a smooth bounded domain $\Omega \subset \mathbb{R}^n$ and for every positive function $u \in C^2(\Omega)$ satisfying the boundary condition $\frac{\partial u}{\partial \nu} = 0$, the following inequality holds

$$\int_{\Omega} |\nabla^2 \sqrt{u}|^2 dx \leq C \int_{\Omega} u |\nabla^2 \log u|^2 dx. \quad (0.1)$$

In the talk, we will discuss applications of this inequality to partial differential equations. Then we will consider generalization of the inequality, focusing on the results obtained in joint work with Tomasz Cieřlak and Wojciech Kryński on analogous functional inequalities on Riemannian manifolds with non-zero curvature.

Finally, we will focus on the constant C in the inequality. We will show that its precise value is relevant in applications and discuss results concerning the optimal constant and its properties.

REFERENCES

- [1] T. Cieřlak, M. Fuest, K. Hajduk, and M. Sieręga, *On the existence of global solutions for the 3D chemorepulsion system*, *Z. Anal. Anwend.*, **43**(2024), 49-65.
- [2] T. Cieřlak, M. Gaczkowski, W. Kryński, *A functional inequality between Hessians in spaces with non-zero curvature*, *Studia Mathematica*, **286**(2026), 303-311.