Singular semilinear elliptic equations in nondivergence form

Artur Rutkowski*

* Wrocław University of Science and Technology, Poland
E-mail: artur.rutkowski@pwr.edu.pl

We study the Dirichlet problem for a singular semilinear equation:

$$\begin{cases} -Pu = \frac{f}{u^\gamma} & \text{in } \Omega, \\ u \equiv 0 & \text{on } \partial\Omega, \end{cases}$$

on a bounded domain Ω, where P is a second-order elliptic differential operator in nondivergence form, $f \in L^1(\Omega)$, and $\gamma > 0$.

We obtain the existence of a solution under the assumptions that $\Omega \in C^{1,1}$ and P has C^1 coefficients, as well as the uniqueness of solutions in $L^1(\Omega)$, under the assumptions that $\Omega \in C^2$ and P has C^2 coefficients. Our proofs are based on a novel combination of tools, such as recently obtained nonlinear versions of Gagliardo–Nirenberg inequalities, estimates of Green functions, and new variants of Kato-type inequalities. Based on a joint work with Agnieszka Kałamajska (Warsaw) and Dalimil Peša (Pardubice).

References

[1] A. Kałamajska, D. Peša, A. Rutkowski. Singular semilinear elliptic equations in nondivergence form. *ArXiv:2605.03704* (2026).