Aggregation–Diffusion Equations with Linear and Nonlinear Diffusion

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I will discuss the existence of weak solutions of the aggregation-diffusion equation

$$\partial_t \rho = \nabla \cdot \Big[\rho \, \nabla \big(U'(\rho) + V(x) + \mathcal{K}[\rho] \big) \Big], \qquad \mathcal{K}[\rho](x) = \int K(x,y)\, \rho(y) \mathrm{d}y,$$

for a strictly convex internal energy U with linear or slow degenerate diffusion, and a symmetric interaction kernel; related to study by Carrillo, McCann and Villani in [1].

Existence is first established on smooth bounded convex domains under a one-sided condition on the combined drift. This condition allows the external and interaction potentials to compensate each other at the boundary. The construction regularizes the diffusion and freezes the nonlocal drift. A time-Lipschitz estimate in the fixed-point iteration provides the time regularity required by parabolic theory. The Riesz–Fréchet–Kolmogorov theorem yields the necessary compactness, allowing us to treat merely integrable initial data. The solution is bounded only when the initial datum is bounded. Whole-space solutions are then obtained by domain exhaustion, using uniform energy and moment estimates.

The talk is based on my Master Thesis written under supervision of Iwona Chlebicka (UW) and David Gómez-Castro (UAM).

References

[1] J. A. Carrillo, R. J. McCann, C. Villani, Kinetic equilibration rates for granular media and related equations: entropy dissipation and mass transportation estimates, *Rev. Mat. Iberoamericana*, **19**(2003), 971–1018.